THEORY OF AUTOMATA

Welcome to !

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Reference Material

1. *INTRODUCTION TO COMPUTER THEORY*, BY DANIEL I. COHEN, JOHN WILEY AND SONS, INC., 1991, SECOND EDITION
2. *INTRODUCTION TO LANGUAGES AND THEORY OF COMPUTATION*, BY J. C. MARTIN, MCGRAW HILL BOOK CO., 1997, SECOND EDITION

Grading

THERE WILL BE ONE TERM EXAM AND ONE FINAL EXAM. THE FINAL EXAM WILL BE COMPREHENSIVE. THESE WILL CONTRIBUTE THE FOLLOWING PERCENTAGES TO THE FINAL GRADE:

MID-TERM EXAMS.	25%
ASSIGNMENTS	25%
FINAL EXAMS.	50%

What does automata mean?

- It is the plural of automaton, and it means “something that works automatically”

Introduction to languages

- There are two types of languages
  - Formal Languages (Syntactic languages)
  - Informal Languages (Semantic languages)

Alphabets

- Definition:

  A finite non-empty set of symbols (letters), is called an alphabet. It is denoted by Σ (Greek letter sigma).

- Example:

  Σ={a,b}

  Σ={0,1} //important as this is the language //which the computer understands.

  Σ={i,j,k}

NOTE:

- A certain version of language ALGOL has 113 letters

  Σ (alphabet) includes letters, digits and a variety of operators including sequential operators such as GOTO and IF

Strings

- Definition:

  Concatenation of finite symbols from the alphabet is called a string.

- Example:

  If Σ= {a,b} then

  a, abab, aaabb, abababababababab

NOTE:

<u>EMPTY STRING or NULL STRING</u>

- Sometimes a string with no symbol at all is used, denoted by (Small Greek letter Lambda) λ or (Capital Greek letter Lambda) Λ, is called an empty string or null string.

The capital lambda will mostly be used to denote the empty string, in further discussion.

Words

- Definition:

  Words are strings belonging to some language.

Example:

If $\Sigma = \{x\}$ then a language L can be defined as

$L=\{x^n : n=1,2,3,\ldots..\}$ or $L=\{x,xx,xxx,\ldots.\}$

Here x,xx,… are the words of L

NOTE:

- All words are strings, but not all strings are words.

Valid/In-valid alphabets

- While defining an alphabet, an alphabet may contain letters consisting of group of symbols for example $\Sigma_1 = \{B, aB, bab, d\}$.
- Now consider an alphabet

  $\Sigma_2 = \{B, Ba, bab, d\}$ and a string BababB.

This string can be tokenized in two different ways

- (Ba), (bab), (B)
- (B), (abab), (B)

Which shows that the second group cannot be identified as a string, defined over

$\Sigma = \{a, b\}$.

- As when this string is scanned by the compiler (Lexical Analyzer), first symbol B is identified as a letter belonging to Σ, while for the second letter the lexical analyzer would not be able to identify, so while defining an alphabet it should be kept in mind that ambiguity should not be created.

Remarks:

- While defining an alphabet of letters consisting of more than one symbols, no letter should be started with the letter of the same alphabet *i.e.* one letter should not be the prefix of another. However, a letter may be ended in the letter of same alphabet *i.e.* one letter may be the suffix of another.

Conclusion

- Σ_1= {B, aB, bab, d}
- Σ_2= {B, Ba, bab, d}

Σ_1 is a valid alphabet while Σ_2 is an in-valid alphabet.

Length of Strings

- Definition:

  The length of string s, denoted by $|s|$, is the number of letters in the string.

- Example:

  $\Sigma=\{a,b\}$

  s=ababa

  $|s|=5$

- Example:

  $\Sigma = \{B, aB, bab, d\}$

  s=BaBbabBd

  Tokenizing=(B), (aB), (bab), (d)

  $|s|=4$

Reverse of a String

- Definition:

  The reverse of a string s denoted by Rev(s) or s^r, is obtained by writing the letters of s in reverse order.

- Example:

  If s=abc is a string defined over Σ={a,b,c}

  then Rev(s) or s^r = cba

- Example:

  Σ= {B, aB, bab, d}

  s=BaBbabBd

  Rev(s)=dBbabaBB

Defining Languages

- The languages can be defined in different ways , such as Descriptive definition, Recursive definition, using Regular Expressions(RE) and using Finite Automaton(FA) etc.

Descriptive definition of language:

The language is defined, describing the conditions imposed on its words.

- Example:

  The language L of strings of odd length, defined over Σ={a}, can be written as

  L={a, aaa, aaaaa,…..}
- Example:

  The language L of strings that does not start with a, defined over Σ={a,b,c}, can be written as

  L={b, c, ba, bb, bc, ca, cb, cc, …}

- Example:

  The language L of strings of length 2, defined over Σ={0,1,2}, can be written as

  L={00, 01, 02,10, 11,12,20,21,22}

- Example:

  The language L of strings ending in 0, defined over Σ ={0,1}, can be written as

  L={0,00,10,000,010,100,110,…}

- Example: The language **EQUAL**, of strings with number of a's equal to number of b's, defined over Σ={a,b}, can be written as

  {Λ ,ab,aabb,abab,baba,abba,…}

- Example: The language **EVEN-EVEN**, of strings with even number of a's and even number of b's, defined over Σ={a,b}, can be written as

  {Λ, aa, bb, aaaa,aabb,abab, abba, baab, baba, bbaa, bbbb,…}

- Example: The language **INTEGER**, of strings defined over Σ={-,0,1,2,3,4,5,6,7,8,9}, can be written as

INTEGER = {…,-2,-1,0,1,2,…}

- Example: The language **EVEN**, of stings defined over Σ={-,0,1,2,3,4,5,6,7,8,9}, can be written as

EVEN = { …,-4,-2,0,2,4,…}

- Example: The language $\{a^n b^n\}$, of strings defined over $\Sigma=\{a,b\}$, as

  $\{a^n b^n : n=1,2,3,\ldots\}$, can be written as

  {ab, aabb, aaabbb,aaaabbbb,…}

- Example: The language $\{a^n b^n a^n\}$, of strings defined over $\Sigma=\{a,b\}$, as

  $\{a^n b^n a^n: n=1,2,3,\ldots\}$, can be written as

  {aba, aabbaa, aaabbbaaa,aaaabbbbaaaa,…}

- Example: The language **factorial**, of strings defined over Σ={1,2,3,4,5,6,7,8,9} *i.e.*

  {1,2,6,24,120,…}
- Example: The language **FACTORIAL**, of strings defined over Σ={a}, as

  $\{a^{n!} : n=1,2,3,\ldots\}$, can be written as

  {a,aa,aaaaaa,…}. It is to be noted that the language FACTORIAL can be defined over any single letter alphabet.

- Example: The language **DOUBLEFACTORIAL**, of strings defined over Σ={a, b}, as

  $\{a^{n!}b^{n!} : n=1,2,3,\ldots\}$, can be written as

  {ab, aabb, aaaaaabbbbbb,…}

- Example: The language **SQUARE**, of strings defined over Σ={a}, as

  $\{a^{n^2} : n=1,2,3,\ldots\}$, can be written as

  {a, aaaa, aaaaaaaaa,…}

- Example: The language **DOUBLESQUARE**, of strings defined over $\Sigma=\{a,b\}$, as $\{a^{n^2} b^{n^2} : n=1,2,3,\ldots\}$, can be written as

  {ab, aaaabbbb, aaaaaaaaabbbbbbbbb,…}

- Example: The language **PRIME**, of strings defined over Σ={a}, as

  {a^p : p is prime}, can be written as

  {aa,aaa,aaaaa,aaaaaaa,aaaaaaaaaaa…}

An Important language

- **PALINDROME**:

  The language consisting of Λ and the strings s defined over Σ such that Rev(s)=s.

  It is to be denoted that the words of PALINDROME are called palindromes.

- Example:For Σ={a,b},

  PALINDROME={Λ , a, b, aa, bb, aaa, aba, bab, bbb, ...}

Remark

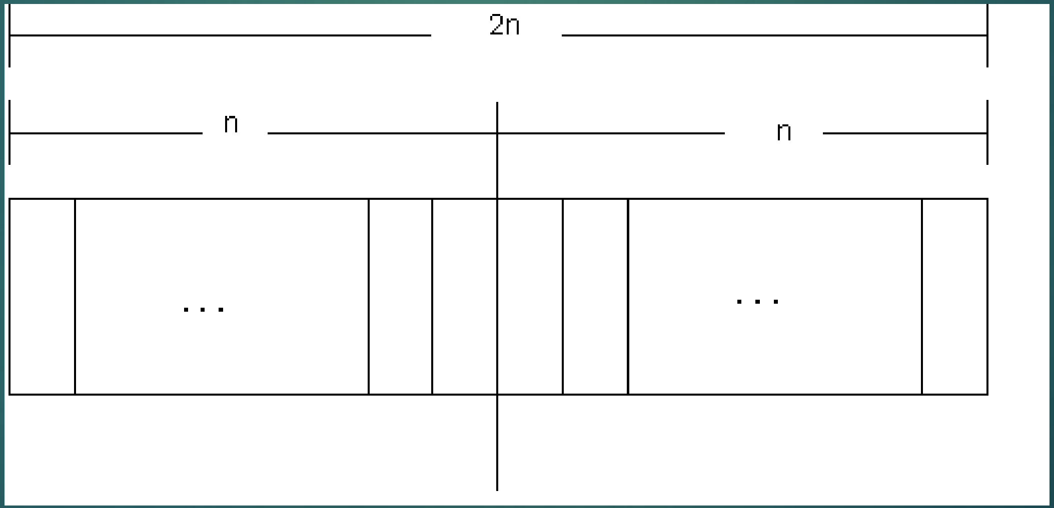
- There are as many palindromes of length 2n as there are of length 2n-1.

To prove the above remark, the following is to be noted:

Note

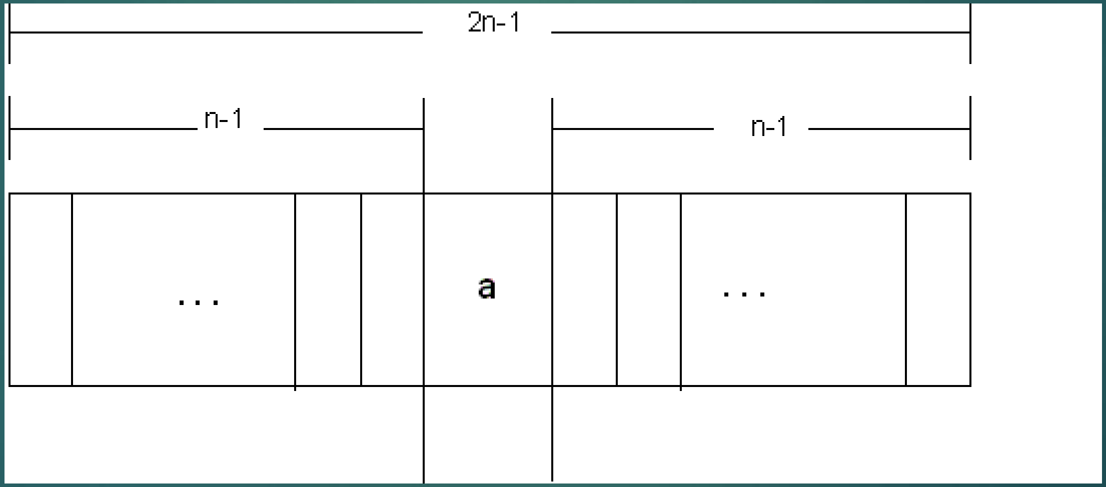
- Number of strings of length 'm' defined over alphabet of 'n' letters is n^m.
- Examples:
  - The language of strings of length 2, defined over $\Sigma=\{a,b\}$ is L={aa, ab, ba, bb} *i.e.* number of strings = 2^2
  - The language of strings of length 3, defined over $\Sigma=\{a,b\}$ is L={aaa, aab, aba, baa, abb, bab, bba, bbb} *i.e.* number of strings = 2^3

- To calculate the number of palindromes of length(2n), consider the following diagram,



which shows that there are as many palindromes of length 2n as there are the strings of length n *i.e.* the required number of palindromes are 2^n.

- To calculate the number of palindromes of length (2n-1) with ‘a’ as the middle letter, consider the following diagram,



which shows that there are as many palindromes of length 2n-1 as there are the strings of length n-1 *i.e.* the required number of palindromes are 2^{n-1}.

Similarly the number of palindromes of length 2n-1, with ' b ' as middle letter, will be 2^{n-1} as well. Hence the total number of palindromes of length 2n-1 will be $2^{n-1} + 2^{n-1} = 2 (2^{n-1})= 2^n$.

Exercise

- Q) Prove that there are as many palindromes of length 2n, defined over $\Sigma = \{a,b,c\}$, as there are of length 2n-1. Determine the number of palindromes of length 2n defined over the same alphabet as well.
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SummingUp Lecture-1

- Introduction to the course title, Formal and In-formal languages, Alphabets, Strings, Null string, Words, Valid and In-valid alphabets, length of a string, Reverse of a string, Defining languages, Descriptive definition of languages, EQUAL, EVEN-EVEN, INTEGER, EVEN, $\{ a^n b^n \}$, $\{ a^n b^n a^n \}$, factorial, FACTORIAL, DOUBLEFACTORIAL, SQUARE, DOUBLESQUARE, PRIME, PALINDROME.